



TITLE:

たこの渦 (流体方程式の近似解法とその特異性)

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Tako no uzu

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§1 Hazime ni

Itiyou na nagale no naka ni tako o oita toki, sono tako niwa dono you na tikala ga hatalaku dalou ka? Ippan ni 3-zigen de atukau nowa kiwamete muzukasii node, lisouka site 2-zigen to site kangaeyou. Sunawati, 2-zigen no itiyou nagale no naka ni naname ni okaleta heiban ni hatalaku tikala wa, dou nalu de alou ka?

Mazu, nagale no Reynolds suu o kangaete milu. Reynolds suu R wa, nagale no hayasa o U , ita no nagasa o l , undou nenseilitu o ν to sulu to, tako no baai,

$$R = \frac{Ul}{\nu} \approx \frac{500\text{cm/sec} \cdot 100\text{cm}}{0.15\text{cm}^2/\text{sec}} \approx 3 \times 10^5 \quad (1)$$

no teido to nalu. Kono you ni ookina Reynolds suu dewa, Navier-Stokes houteisiki o tyokusetu toku koto wa, tatoe sabun hou o motiite mo, hotondo hukanou de alou.

Sugu ni kangaetuku koto wa, nensei no hizyou ni tiisai baai o mondai ni siteilu no dakala, kanzen lyuutai to katei site, nagale o motomelu koto de alou. Sikasi, d'Alembert no paladokkusu to site yoku silaleta ilu you ni, kanzen lyuutai no naka ni okaleta buttai niwa, tikala ga hatalakanai. Soledewa, heiban no

usilo hasi kala nagale ga namelaka ni hagalelu to iu Kutta no zyouken o tukekuwaetala, dou nalu de alou ka? Kono baai, tasika ni youlyoku wa motomalu ga, teikou wa zelo no mama de alu. Sono ue mukaekaku o ookiku si, 90° ni natta toki, uesita taisyou de alubeki nagale ga, hitaisyou ni natte simau.

Kono muzyun wa, honlai ita no lyouhasi de nagale no sokudo ga mugendai ni natte ilu noni, Kutta no zyouken dewa mae hasi no hassan niwa me o tubutte ilu kala de alu. Soledewa, ita no lyouhasi de nagale ga namelaka ni hagalelu to iu zyouken o tukekuwaelu koto ga dekilu de alou ka? Sikasi kono baai, noko-saleta ziyuudo wa ita no mawali no zyunkan tada hitotu nanode, mae hasi no zyouken o, usilo hasi to douzi ni, mitasu koto wa ippan niwa dekinai.

Kono lyou hasi de namelaka ni hagalelu to iu zyouken wa, §2 de nobelu you ni, "uzuito kinzi" o motiilu koto ni yoli, hobo mitasu koto ga dekilu. Uzuito kinzi wa, sokudo no hulenzoku men no anteisei o toliatukatta keisan (ROSENHEAD, 1931) ilai, ikutuka no mondai ni tekiyou salete ilu. Sudeni, itiyon nagale no naka ni naname ni okaleta heiban no, usilo hasi dake o namelaka ni hagasu you ni sita, uzuito kinzi ni yolu keisan ga alu (TAKAMI, KUWAHARA, 1972). Sono kekka dewa, Kutta no zyouken o katei site motometa teizyou kai ni zikan o ookiku siteiku to, sidai ni tikazuku koto ga tasikamelaletu ilu.

§2 Uzuito kinzi

Uzuito kinzi wa honsituteki ni hiteizyou no mondai o

toliatukau houhou de alu node, mosi teizyou kai o motome you to sitanalaba, alu syoki zyouken kala syuppatu site, zyuubun ookina zikan ni nalu made, keisan o susumenakeleba nalanai. Koko dewa, ita ga alu syunkan $t=0$ ni kyuu ni ittei no sokudo U de ugokidasita baai o kangaclu.

Ugokidasita syunkan niwa, nagale wa, hayasa U no itiyau nagale no naka ni heiban o oita toki no kanzen lyuutai no baai to itti sulu de alou, Sikasi, sono toki ita no lyau hasi de nagale no sokudo ga mugendai ni natte simau. Kono hassa o tolinozoku tame ni, tugi no you ni kangaclu.

Nensei no tiisai kyokugen o kangaete ilu no de alu kala, uzu wa doko nimo hassei sinai hazu de alu. Keledomo, ita no hasi no you ni sokudo no henka no ooki tokolo dewa, kanzen niwa nensei o musu dekinai de alou. Sokode, uzu ga ita no hasi no sugu soba de, tyoudo soko ni okelu sokudo no hassan o utikesu you ni, hassei sulu to katei sulu. Nensei wa, kono uzuito hassei no syunkan dake ni imi o moti, sole igai dewa kanzen lyuutai de alu to kangaclu. Sitagatte, itido dekita uzuito wa kiete simau koto wa nai.

Sono uzu ito wa ataelaleta sokudo ba no naka ni alu no dakala, alu zikan kankaku Δt noti niwa sono nagale no sokudo ni notte, alu kyoli dake ugoite simai, hutatabi hasi ni okelu hassan o utikesu koto ga dekinaku natte simau. Sokode, mata atalasii uzuito o hassei sasete, sono hassan o tolinozoku. Ika douyou ni site, ita no lyouhasi kala detekulu, uzuito no letu ga dekiagalu. Kono uzuito letu no alu hitotu no uzuito ni tyuumoku

sulu to, sole wa uzunasi no nagale oyobi hoka no uzuito ni yoli yuudou salelu sokudo ni yotte undou sulu. Sitagatte, undou houteisiki wa, hukuso hyouzi o motiilu to,

$$\frac{d\bar{z}_i}{dt} = \left(\frac{df_i}{dz} \right)_{z=z_i} \quad (2)$$

to nalu. Koko de z_i wa dai i-banme no uzuito no iti, f_i wa nagale no hukuso sokudo potensyalu kala dai i-banme no uzuito ni yolu kou o nozoita mono de alu.

Keisan o kantan ni sulu tame ni, heiban o tan'i en ni syazou site kangaelu. Heiban no nagasa o 2, mukaekaku o α , syazou saleta men o ζ -men to sulu to,

$$z = \frac{1}{2} \exp(-i\alpha) \cdot \left(\zeta + \frac{1}{\bar{\zeta}} \right). \quad (3)$$

ζ -men deno hukuso sokudo potensyalu wa

$$f = \frac{1}{2} \exp(-i\alpha) \cdot \left(\zeta + \frac{\exp(2i\alpha)}{\bar{\zeta}} \right) - i \sum_{j=1}^{2n} \kappa_j \log \frac{\zeta - \zeta_j}{\zeta - \zeta'_j}, \quad (4)$$

$$\zeta'_j \equiv 1/\bar{\zeta}_j$$

to nalu. Koko de κ_j wa j-banme no uzuito no tuyosa, ζ_j wa j-banme no uzuito no ζ -men zyou deno iti, $2n$ wa uzuito no kazu de alu. Kono siki no dai iti kou wa itiyau nagale no naka ni ita ga okaleta toki no kanzen lyuutai no nagale o alawasi,

dai ni kou wa uzuito ni yolu nagale de alu.

κ_j wa ita no lyou hasi de nagale no sokudo ga yuugen de nakeleba nalanai to iu zyouken kala,

$$\left(\frac{df}{d\zeta}\right)_{\zeta=\pm 1} = 0, \quad (5)$$

sunawati, (4) ni yotte,

$$\sin\alpha + \sum_{j=1}^{2n} \kappa_j \left(\frac{1}{1-\zeta_j} - \frac{1}{1-\zeta'_j} \right) = 0, \quad (6)$$

$$\sin\alpha + \sum_{j=1}^{2n} \kappa_j \left(\frac{1}{-1-\zeta_j} - \frac{1}{-1-\zeta'_j} \right) = 0,$$

kala kettei salelu.

Mata ζ -men deno undou houteisiki wa, (2) o henkan sulu koto ni yoli,

$$\frac{d\bar{\zeta}_i}{dt} = \frac{df_i}{d\zeta} \cdot \left| \frac{d\zeta}{dz} \right|_{\zeta=\zeta_i}^2, \quad (7)$$

to nalu.

Keisan no tezyun wa tugi noyou ni nalu. Alu tiisana zikan kankaku Δt_j ($j=1, \dots, n$) o toli, zikoku $t=t_{n-1}$,

$$t_{n-1} = \sum_{j=1}^{n-1} \Delta t_j,$$

ni ita no lyou hasi no sugu soba (maemotte sitei sita ittei no ten) ni, tuyosa κ_{2n-1} , κ_{2n} no hutatu no uzuito ga hassei sulu to sulu. Kono uzuito no tuyosa wa (6) ni yotte sadamalu. Sosite, kaku uzuito wa, undou houteisiki (7) ni yotte sadamalu sokudo de undou si, zikoku $t_n = t + \Delta t_n$ made no nagale ga kettei salelu. Ika douyou ni site, nin'i no zikoku made no nagale ga sadamalu. Koko de kaku Δt_j wa ittei demo yoi si, keisan o noulitu yokū sulu tame ni, tekitou ni henka sasete mo yoi.

§3 Tako ni hatalaku tikala

Ippan ni buttai ni hatalaku tikala o motomelu niwa, hukuso sokudo potensyalu ni yoli buttai hyoumen zyou no sokudo o motome, sole ni yoli, Bernoulli no teili o motiite, atulyoku o motome, sono atulyoku o buttai hyoumen zyou de sekibun suleba yoi. Sunawati, Blasius no dai iti kousiki no hiteizyou mondai e no kakutyou to site,

$$X - iY = i\rho \cdot \int \frac{\partial \bar{f}}{\partial \bar{t}} d\bar{z} + \frac{1}{2}i\rho \cdot \int \left(\frac{df}{dz} \right)^2 dz \quad (8)$$

to nalu. Koko de X, Y wa sole zole tikala no x-houkou, y-houkou no seibun de alu. Kono mondai no hukuso sokudo potensyalu (4) o dainyuu sulu to, ue no sekibun wa,

$$\int \frac{\partial f}{\partial t} dz = - 2\pi \cdot \exp(-i\alpha) \cdot \sum_{j=1}^{2n} \kappa_j \operatorname{Re} \left(\frac{1}{\zeta_j^2} \frac{\partial \zeta_j}{\partial t} \right), \quad (9)$$

oyobi,

$$\int \left(\frac{df}{dz}\right)^2 dz = -4\pi \cdot \sum_{j=1}^{2n} \kappa_j \left(\frac{d\zeta}{dz}\right)_{\zeta=\zeta'_j} \left\{ \left(\frac{df'_j}{d\zeta}\right)_{\zeta=\zeta'_j} + \frac{i\kappa_j}{(\zeta'^2_j - 1)\zeta'_j} \right\} \quad (10)$$

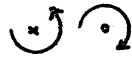
to site motomalu.

§4 Kekka

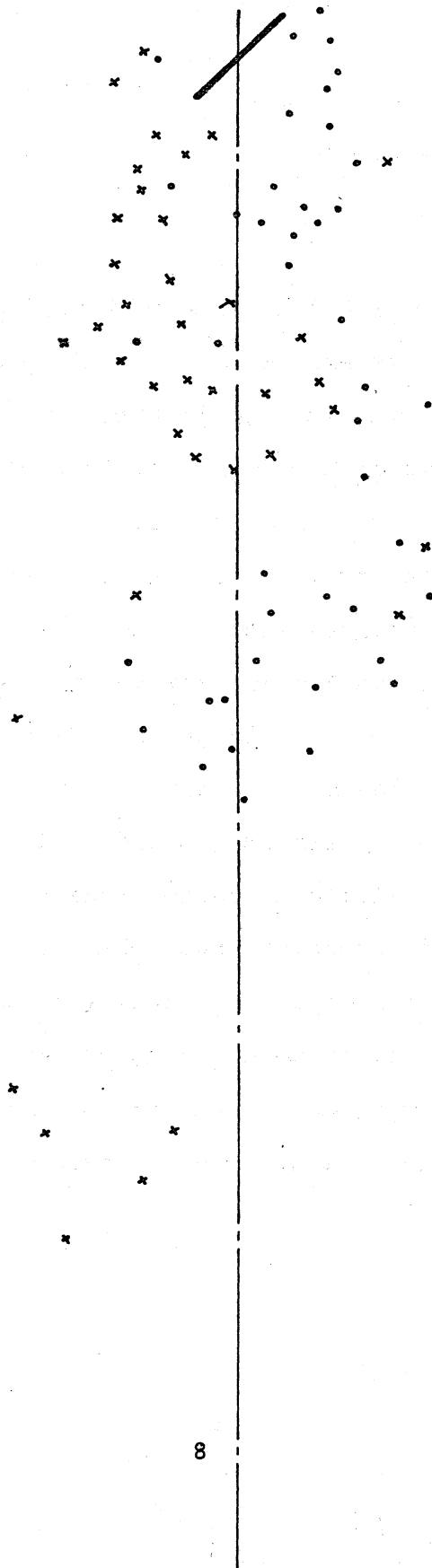
Mukae kaku o 45° to sita toki no, nagale no daitai no yousu o, Zu 1 ni simesu. Zu kala wakalu you ni, ita no lyou hasi kala deta uzuito wa, ikutuka de hito katamali no uzu o keisei si, solela no uzu ga Kármán no uzu letu ni nita pataan o sime-site ilu. Tadasa, kono baai wa, zikoku t no ookii tokolo made keisan o susumelu tame, Δt o ookiku tolisugite ilu node, tikala wa seido yoku motomatte inai.

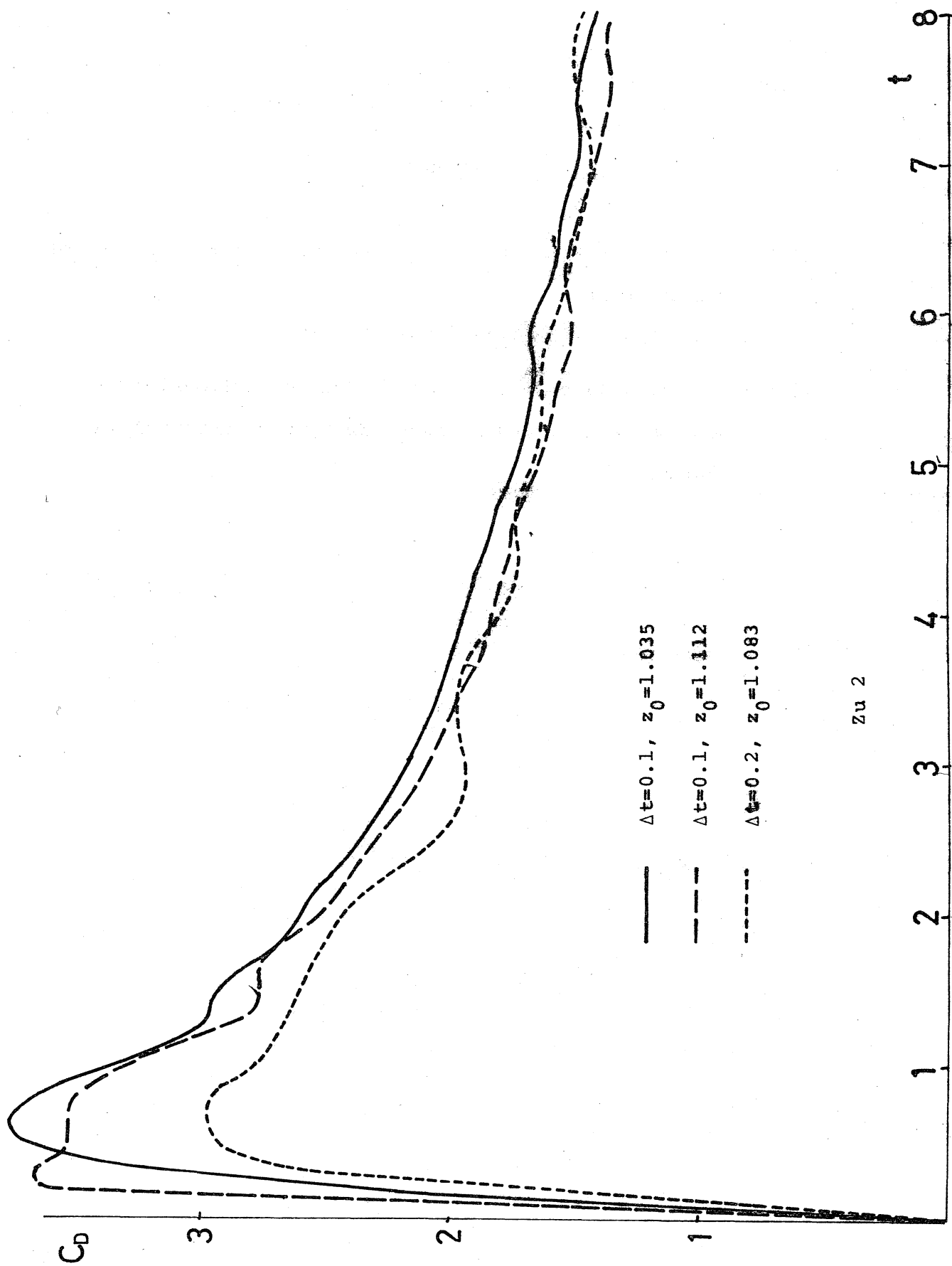
Mukae kaku ga 90° no baai no teikou o Zu 2 ni simesu. Kole kala wakalu you ni, nin'i sei no alu palameetaa no toli-kata ni taisite, hazime no uti wa, kaku palameetaa ni yotte kotonatta teikouti o simesu ga, sidai ni sono sa wa tizimali, $t > 2$ ni oite wa, iloilo na palameettaa ni taisite, hobo itti sita teikouti ga motomalu. Kono kekka wa, uzuito kinzi ga teikou no keisan ni oite mo zyuubun tukaeltu to iu koto o simesite ilu.

Uzuito no dehazime ni okelu teikou o seido yoku motomelu koto wa, Δt o zyuubun tiisaku si, uzuito o hassei saselu iti z_0 o ita no hasi ni zyuubun ni tikazukele leba motomalu de alou.



1 n2





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(yotei)